# EXECUTIVE SUMMARIES



An Executive Summary consists of a brief description of the problem followed by a largely equation-free summary of the progress made and the results obtained by the study group.

# SUSTAINABLE TOURISM: CAPPING VISITORS NUMBERS

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**Executive Summary**

The rise of over-tourism or mass tourism has become a pressing issue in numerous destinations. When the number of visitors exceeds the carrying capacity of a destination, it can lead to overcrowding, strain on local infrastructure, cultural degradation, and a decline in the quality of life for both residents and visitors. Over-tourism has gained attention as a global and regional challenge, as it affects the sustainability and livability of the destination. Protecting sensitive environments, providing a more enjoyable visitor experience, and preserving natural and cultural resources are necessary.

It is vital for natural attractions to sustain the physical and ecological balance of visitors. Thus, certain tourist attractions strongly advocate limiting the number of visitors per day. Determining the optimal number of visitors that can be accommodated without compromising the visitor experience or the tourism attraction is crucial. The main challenge is determining a particular tourist attraction's social carrying capacity (SCC). The primary objective of the study group was to develop a viable mathematical model to determine the social carrying capacity of a tourist attraction to mitigate the negative impact of over-tourism while

providing a high-quality experience for visitors. The model should consider the available infrastructure, activities, natural and cultural resources, and accommodation. The tourist attractions in consideration are Manyeleti Nature Reserve, Mariepskop Nature Reserve, Bushbuckrigde Nature Reserve, and Injaka Dam.

The problem is incredibly challenging, and the solution to the problem will be of global importance to the tourism sector. The study group proposed a model to determine carrying capacity by simulating the visitor flow of a given tourist attraction. The model is derived from traffic flow problems. A traffic light network is analogous to a network of different attractions (activities). The model considers the following parameters for a particular tourist attraction: the arrival rate, length of stay, activities, and attraction capacity. The aim is to avoid long queues for any activity (attraction) in the network. Through simulations, we can determine the carrying capacity, which is a point at which long queues are unavoidable in a network. The derived and proposed model is straightforward and immensely useful. It will require data to simulate real-world situations and identify key model parameters.

The carrying capacity alone will not necessarily translate to a high-quality visitor experience. The study group determined that the mathematical model would be insufficient to have a real impact in mitigating the challenges mentioned earlier. In addition to the model, we consider the following points in an attempt to find a wholesome solution to the problem:

- Impact assessment on the local community: This requires assessing how these tourist attractions contribute to improving the communities they operate in. This will be assessed through their social projects, outreach programmes, staff compliment, etc.

- Optimization of visitor satisfaction: This will entail formulating an optimization problem that maximizes the community's visitor satisfaction and economic benefit while minimizing the negative impact of overcrowding, resource depletion, and cultural degradation.

- A functional management system: We need a regular carrying capacity assessment incorporating visitor feedback on the model to improve overall visitor satisfaction. Thus, continuous strategic management based on evolving conditions will be paramount.

Participants in the study group could only develop the basic model to determine the carrying capacity. We acknowledge that more work still needs to be carried out to solve the over-tourism problem. We need to obtain real data from these tourist attractions to validate the proposed model. Also, we still need to extend the model to input the other vital parameters such as available infrastructure, size of these attraction sites, natural and cultural resources, available accommodation, types of accommodation, season (time of the year), etc. Ideally, we want to eventually advise these nature reserves on the exact number of visitors allowed in a particular park at a specific time.

# SUGAR FACTORY CONTROL STRATEGY

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**Executive Summary**

A sugar factory consists of unit operations in series interspersed with buffer tanks and storage. A buffer is a unit where the volume is exploited to provide smoother operations. These operations are extraction, clarification, evaporation, and crystallization. The operations must be controlled so the buffer tanks neither run empty nor overflow. The sugar cane that is processed comprises fiber, brix, and water. The exact amount of each component depends on the variety of cane, time of the year, and growing conditions. The throughput of each unit operation is limited by at least one of these components. In some units, a minimum throughput is required to maintain performance. At different times of the year, the operations experience bottlenecks, which depend on the composition of the cane. Under optimum conditions, the unit operation that is the bottleneck at the time will run at or near capacity. The other operations will be controlled to keep the level in the buffers from running dry or overflowing. The objective is to construct the optimal control strategy for the whole process to run smoothly without any units running dry or overflow.

The problem was addressed in two approaches, each trying to answer one of the following questions. Firstly, how do we minimize the residence time, limited by the buffer tank capacity required to accommodate the normal fluctuations in operations? Second, can the buffer tank theory be applied to the more complex multi-component streams in sugar factories where the bottlenecking component changes along the process?

To address the first question, a previous investigation identified that the bottleneck is more likely to occur at the final processing unit, the sugar crystallization. Crystallization is when liquid syrup is fed into pans and heated to form sugar crystals. In the MISG, we developed a simple single-compartmental model to control the syrup's feeding rate into the tank. The objective was to maximize the throughput of the sugar crystallization unit while keeping the syrup tank at a certain level. The resulting model was a constrained optimal control problem, which proved difficult to solve. Alternatively, one could build the optimal control for each operational unit and join all the control problems into a chain. It became clear that, ultimately, one must control the rate at which sugar cane is fed.

All the operational units preceding the sugar crystallization were bundled to form a single compartment. As such, the whole process is viewed as two tanks in a series. Thus, we modeled the process as a coupled tank system with three valves. The first tank, formed by concatenated units, had an input valve that characterized the rate at which sugar cane was fed into the system. The first tank output valve fed directly to the second tank, the sugar crystallization unit. The sugar crystallization had an output valve that characterized the rate of sugar production. We assumed steady, non-viscous, and incompressible fluid flow. We then constructed an optimal control model. Here, the objective was to find an optimal control of the sugar cane feeding rate such that the fluid level in the sugar crystallization stage is as close as possible to the steady state level. The numerical results indicate the optimal control should be non-negative and that sugar levels in the two tanks should always be non-negative to ensure a smooth flow in the production process.

To address the second question, we used the buffer tank theory to track sucrose (sugar) concentration throughout the sugar production. A simple system of coupled linear differential equations describes changes in sucrose concentration. Ideally, we solved an optimization problem with estimated inputs and observed values to identify model parameters. However, we did not have the sucrose concentration data for the study group. Thus, we estimated most of the parameters from past reports in the literature. Results indicate that as water and mud are removed during the different processes, the sugar concentration increases. This is consistent with what is observed in the actual sugar production, where it is observed that sucrose concentration is greatest in the final crystallization stage.

Lastly, we acknowledge that more work still needs to be carried out in an attempt to solve this problem. We need to obtain real data from sugar production to validate the derived models. Also, we still need to factor in the residence time, which is the hold-up time of materials in the buffer tanks, in the buffer tank theory. A possible merge of the two considered approaches should be explored in order to be able to simultaneously control disturbances and bottlenecks in the sugar production process while at the same time ensuring that we extract as much sucrose from every sugar cane fed into the system.

# ENERGY MANAGEMENT OPTIMIZATION FOR WATER DISTRIBUTION NETWORK TANKS

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**Executive Summary**

Water treatment and distribution is one of the most energy consuming processes in the world. For example, the percentage of energy usage in water treatment and distribution in Spain is 7% of all energy consumed in Spain. Optimizing energy in water distribution is therefore a priority. Water consumption varies throughout the day. Energy prices to run pumps also varies throughout the day. For example Barcelona has a tariff in which the energy costs vary with three different prices based on three different time slots. The problem is to optimise the pump operation schedule of the water distribution network such that operation costs are minimised while maintaining adequate water storage levels. The water consumption of all consumers throughout the day is assumed known. The acceptable maximum and minimum water tank levels are given. It is assumed that the water flow is instantaneous and that all pumps operate at the same wattage and flow rate and that all tanks have the same physical dimensions. A single tank model was first considered. The study group randomly generated a set of data to check and validate the effectiveness of the model. The full network was then considered which is a mixed integer linear programming problem. Small instances of this type of problem can be solved using MS Excel or the optimization tool box of Matlab. Larger instances will require the use of more sophisticated optimization solvers such as CPLEX or GUROBI.

# ADSORPTION OF MULTIPLE CONTAMINANTS FROM A FLUID STREAM


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**Executive Summary**


One practical method of removing fluid based environmental contaminants is column sorption, either through absorption or adsorption. Mathematical models of column adsorption focus on the removal of a single contaminant, in which case the process is defined by an advection- diffusion equation linked to a sink model, typically an ODE, determined by the chemistry. The analysis of single contaminant models is largely based on asymptotic reductions and travelling wave solutions, see [1,2] for example.

Adsorption is particularly effective at the source of pollution, for example at a chimney outlet or exhaust, where concentrations are high. Exhaust gases are usually composed of multiple components and their capture is significantly more complex than with a single component: molecules will compete for available adsorption sites as well as displace previously adsorbed molecules. This is not described by current mathematical models. The focus for the study group was then to develop suitable models for multiple contaminant fluids and to seek solution methods.

The first step undertaken by the group was to analyse possible chemical reactions and translate these to an ODE form. For two contaminants a model was written down which had both components competing for sites, standard desorption and the second molecule displacing the first. These were linked to an advection-diffusion equation for each species. Non- dimensionalisation and substitution of typical values indicated that as a first approximation terms involving the Dahmkohler and inverse Peclet numbers could be neglected.

It was demonstrated that, due to the interaction between the two contaminant waves moving at different speeds, a travelling wave solution could not be found. The group then focused on numerical solutions.

For the two component problem a numerical solution was developed and parameters adjusted manually such that the outlet concentrations resembled the form of the experiments. A four component numerical solution was also developed which showed qualitatively similar outputs to the experiments reported in [3].

In the future it is intended to analyse the two component model and correctly identify all unknowns - it is possible that the chemistry will show relations between the current unknowns and so reduce the parameter space. The fitting process will also be formalised. The extension to multiple contaminants will be dealt with, but it must be noted that more care must be taken regarding the competition: cases may exist where components displace each other at different rates or only certain components carry out the displacement. This suggests a wide variety of behaviour and the need to deal with multiple contaminant adsorption in a systematic way.

**References**


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# MODELLING AND OPTIMIZATION IN THE ROBO CUP DOMAIN

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**Executive Summary**

The Wits Robo Cup Team was founded in 2019 with the aim of developing a sustainable and competitive team to compete in the annual international Robo Cup robotics soccer competition. RobCup is a platform for testing learning scenarios where multiple skills, decisions and controls have to be learned by a single agent and agents have to cooperate and compete in the game of soccer. Each robot has 22 degrees of freedom: six in each leg, four in each arm and two in the neck. The position of the ball is known throughout. The study group was set two problems. The first problem was to optimise a set of parameters to maximise the distance a ball is kicked. The parameters of the leg, foot, knee and hip were optimised in a formula for the distance kicked using a linear simplex method but there were difficulties with the constraints because it is a dynamical problem. The second problem was, given the set of all player positions, teammates and opponents, and the ball position, to optimise positions to minimise defensive threats and maximise counter-attack opportunities. The study group investigated this problem using a formula for the most important position in the field given the position of the ball and of two opponents.

# TEMPERATURE MODELLING IN A FURNACE


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**Executive Summary**

## 1 Motivation

The smelting of Platinum group metals (PGM) is conducted in a violently disturbed, high temperature container fueled by coal and oxygen. The furnace is very difficult to examine while in operation due to the excessive temperatures and dangerous environment. The group was requested to present some possible models for the process and to predict the temperature in the furnace during operation. An issue of primary concern is stability. Eruptions can occur causing very dangerous conditions and hampering production. An improvement in understanding of the process is desirable.

## 2 Process

The ore is first pre-processed using a flotation device, removing a lot of unwanted materials. The remaining ore concentrate is then placed in a furnace and heated to a very high temperature by an exothermic chemical reaction which is sustained by the introduction of hot air, coal and oxygen via a lance. Liquid metallic sulphide and other dense metal droplets are formed in the slag and, because of their high density (SG - Specific Gravity = 4.5) compared to the ore(gangue) (SG=3.5) drift down into the matte layer. The matte layer consists mainly of copper, iron, and iron sulphides together with very small amounts of the Platinum Group Metals (PGM) (see Figure 1). Remaining, lighter, material stays in the slag layer at the top. The gas from the lance and the sulphur dioxide produced by the reaction fluidises the slag and this is the main initiator and controller of the process.

The volume ratio of the component of the processed ore that enters the matte layer to that of the slag layer is typically 1 to 4-9. Slag and matte layers are tapped at different times from different heights to extract the product and waste and to keep the process stable.

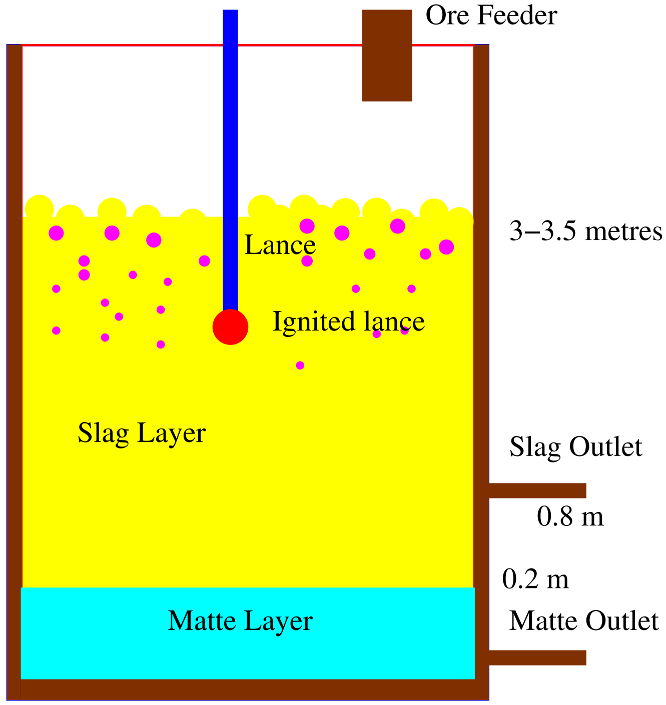


Figure 1: Platinum furnace

The extracted metals and hence the matte layer are much more dense than the slag layer above. As a consequence, there will be a strong density stratification between the two layers, so the matte layer will be extremely stable. On the other hand, the upper layer is highly agitated.

A considerable amount of gas is generated during the process and these gases generate a significant amount of fluidisation in the slag, a desirable feature of the process. The matte layer is relatively thin ($\approx$ 20cm) and motionless, while the slag layer cycles from 3.5-1m in depth as it is replenished and then tapped.

The tapping process can lead to rapid changes in height and hence volume, potentially causing instabilities. Other causes of potential mismatches between heat input and heat absorption are variable feed quality of coal, misjudgment of the furnace state by the operator and errors in lance height, In particular, the chemical reactions may lead to thermal runaway if the slag gets too hot, and could result in a violent fluidisation and increase in volume of the slag layer (like the sudden overflowing of boiling milk).

## 3 Models

During the week the group considered a number of different factors. First was an analysis of the chemical reactions in the process and the thermodynamical processes. The production of gas within the furnace is an important component in the generation of the turbulence and splashing, and so a model of global gas production from the process was developed. The combination of the chemistry and gas production provides a sound basis for an examination of the conditions in the furnace. The aim is to predict the onset of foaming in which the gaseous fraction suddenly expands. It is possible that this fluidisation could result in thermal runaway which takes place over very short time and length scales so we might regard the slag/gas as a fluidised bed with combustion with the possibility of thermal runaway occurring at the hottest point.

As a start a simple model was written down in which the whole slag layer is assumed to be completely mixed and the heat is averaged across the whole region. Heat input from the lance and released from the chemical reaction, losses through phase changes, conduction through the boundaries and bubbling out through the top must add up to zero if the system is in equilibrium. This information could be used to determine the changes in heat balance if the layer depth changes and requirements for adjusting lance inputs, for example, during tapping of slag. Also a spatially-dependent model was written down which incorporated heat conduction assuming an axisymmetric furnace.

## 4 Final comments

A fluidisation and combustion model should allow a determination of the potential for conditions of thermal runaway and violent fluidisation.

The tapping process seems to have strong potential to cause such instabilities, and so a preliminary suggestion is that more frequent, but smaller tapping of the slag layer will reduce the likelihood of such an instability and retain more consistent operating conditions. Less variation in slag level would require less adjustment of lance height and feed rate. Finally the group felt that more detailed information about the process and real data would be required for a more detailed analysis and to identify what instabilities are important and how they might arise. An important start to any future work would be a survey of the theory of combustion and fluidised beds.